Final Exam Review - Day 3 - 12/8/2023

Final Exam

Posted Dec 4, 2023 10:29 AM

The Final Exam will be on Friday, December 15 from 10:30am-12:30pm. We will all be in Loeb Playhouse (STEW 183). When you enter the lobby of Loeb, you will need to pick up a lapboard for taking the exam. You will be on either the main floor or the balcony, depending on your instructor.

Main Floor: Victor Hughes, Jakayla Robbins, Alexandra Cuadra

Balcony: Ben Doyle, Dave Norris

There are no old published finals available, but there is plenty of practice in LON-CAPA. The material on the final exam is evenly distributed among the topics covered this semester. There is an Exam Memo posted in Contents. On the second page of the exam memo is the formula sheet that you will be given on the exam (it will be attached at the back of the exam).

Attachment(s):

- FinalExamMemo.pdf (49.88 KB)
- 16020_final-f23_merged.pdf (286.66 KB)

Office Hours during finals week:

* By appointment only
Send email to Dr. Robbins by 6AM on Wed 12/13.

Formula Sheet - MA 16020 Final Exam

Geometric Series:

The geometric series $\sum_{n=0}^{\infty} ar^n$ with common ratio r converges if $|r| < 1$ with the sum

$$S = \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

Power Series/Maclaurin Series:

$|x| < 1$

$$\begin{aligned}
\frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots \quad |x < 1| \\
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \\
\sin x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \\
\cos x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \\
\ln(1+x) &= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots
\end{aligned}$$

Second Derivative Test

Given the critical point (a, b), such that $f_x(a, b) = 0$ and $f_y(a, b) = 0$, and let

$$D(a, b) = f_{xx}(a, b)\, f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

- If $D > 0$ and $f_{xx}(a, b) > 0$ then $f(a, b)$ is a relative minimum.
- If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a relative maximum.
- If $D < 0$, then $f(a, b)$ is a saddle point.

Series:

geometric series $a + ar + ar^2 + ar^3 + \cdots\cdots = \sum_{n=0}^{\infty} ar^n$

Most students find it helpful to take Σ and write out like this.

but there are other Σ reps.

(T) $a + ar + ar^2 + \cdots\cdots = \frac{a}{1-r}$ if $|r| < 1$

1st / 1−ratio

diverges if $|r| \geq 1$

Power series think "infinite polynomial"

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots\cdots = \sum_{n=0}^{\infty} a_n x^n$$

One type of power series comes from a geometric series.

$$\underbrace{1 + x + x^2 + x^3 + \cdots}_{\text{geometric}} = \sum_{n=0}^{\infty} x^n$$

geometric $a = 1\text{st term} = 1$

$r = \text{ratio} = x$

So $1 + x + x^2 + x^3 + \cdots\cdots = \frac{1}{1-x}$ if $|x| < 1$

We often take things of the for $\frac{\quad}{A \pm Bx^k}$

and say that this can be made to look like sum of a geometric series.

Find a power series representation for the following function and the radius of convergence.

$$f(x) = \frac{6}{10 + x^2}$$

I want f(x) to look like $\frac{a(x)}{1 - r(x)} = a(x) \cdot \frac{1}{1 - r(x)}$

Goal

$$\frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n$$

$$f(x) = \frac{6}{10 + x^2} = \frac{1}{10} \cdot \frac{6}{1 + \frac{x^2}{10}}$$

$$= \frac{6}{10} \cdot \frac{1}{1 + \frac{x^2}{10}}$$

$$= \frac{3}{5} \cdot \frac{1}{1 - \left(-\frac{x^2}{10}\right)} \quad \frac{1}{1 - r}$$

$$= \frac{3}{5} \sum_{n=0}^{\infty} \left(\frac{-x^2}{10}\right)^n = \sum_{n=0}^{\infty} \frac{3}{5} \frac{(-1)^n x^{2n}}{10^n}$$

If the did not simplify $\frac{6}{10}$, then they might write it as

$$\sum_{n=0}^{\infty} \frac{6}{10} \frac{(-1)^n x^{2n}}{10^n} = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 6 \cdot x^{2n}}{10^{n+1}}$$

Radius of convergence

$|r| < 1$

Solve for $|x| < R$ ← radius of convergence

For this problem

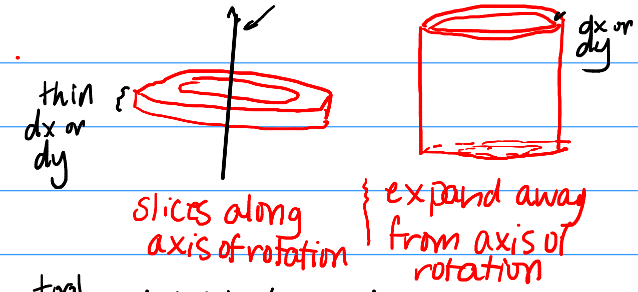
$|r| < 1$

$\left|\frac{-x^2}{10}\right| < 1$

$\frac{|-1| \cdot |x^2|}{|10|} < 1$

$$\frac{|x^2|}{10} < 1$$
$$|x^2| < 10$$
$$|x|^2 < 10$$
$$|x| < \sqrt{10}$$
radius of conv $= \sqrt{10}$

One final look @ volumes.



Volume Review.

Disks: $V = \int_a^b \pi r^2 \; dx/dy$

Washers: $V = \int_a^b \pi (R^2 - r^2) \; dx/dy$

Shells: $V = \int_a^b 2\pi \underset{\uparrow \text{ easy}}{r} \; \underset{\uparrow \text{ harder}}{h} \; dx/dy$

axis \ tool	Disks/Washers	Shells
vertical (like y-axis $x=0$)	dy	dx
horizontal (like x-axis $y=0$)	dx	dy

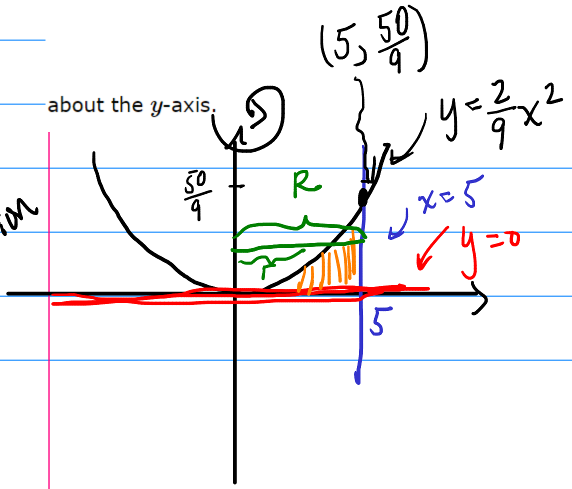
Test Review, lesson 15 #3

Find the volume of the solid generated by revolving the region enclosed by the curves

$$y = \frac{2}{9}x^2,\ x = 5,\ \text{and } y = 0$$

about the y-axis.

All radii go out from the axis of rotation



Washers

@ y-axis (vertical axis) use dy

R: $x = 5$

r: $x = \sqrt{\frac{9y}{2}}$

Coming from $y = \frac{2}{9}x^2$

Set up w/ washers:

$$V = \int_0^{50/9} \pi \left(5^2 - \left(\sqrt{\frac{9y}{2}}\right)^2 \right) dy$$

$$= \int_0^{50/9} \pi \left(25 - \frac{9y}{2}\right) dy$$

but needs to be written as

$x = \underline{f(y)}$

$\frac{9}{2}y = x^2$

$x = \pm\sqrt{\frac{9}{2}y}$

right half of parabola

$x = \sqrt{\frac{9}{2}y}$